Exploration of Chaotic Orbits Using the Lyapunov Exponent in the Restricted Three-Body Problem

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Dynamical systems are mathematical models that are used to describe the laws of a natural system. One example of such is the Duncan’s Map which models the Three-Body Problem that entails three particles. The primary and the secondary particle have mutual gravity acting on each another while the “test” particle has a small mass where its gravitational pull does not affect the primary or secondary particle. In my project, I varied the parameters of this model which are the Jacobi constant and the mass of the secondary particle to see if chaotic orbits appear. Numerical approaches on identifying chaos have yet to be implemented, the Lyapunov exponent was used to quantitatively identify chaotic orbits. Chaotic orbits appeared when the Jacobi constant was decreased, and when the mass of the secondary particle increased. Some of the unstable and stable regions coincided with the presence of mean-motion resonances. Also, Duncan’s criteria of chaos resulted in a 2/7 power law that predicts the onset of chaos, which was further probed using the Lyapunov exponent. This is remarkable because a unified definition of chaos is not yet found. Overall, this research is important because it could lead to effective space mission design and give one a glimpse to the complex past and future of the solar system. Further work could include changing the model to allow for dissipation, large eccentricity of the secondary particle and testing the 2/7 power law with other chaos indicators such as the correlation function or the Kolmogorov-Sinai entropy.