Attempting To Define Tetration of Non-Integer Heights

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Tetration is a fundamental mathematical operation that follows in the sequence of addition, multiplication, and exponentiation. x^^n is defined as x to the power of itself, n times. Naturally, the question arises: What happens if n is a non-integer? Many generalizations to non-integers have been produced over the history of tetration, all of which fall short in one way or another. As such, there is no universally accepted definition of non-integer tetration. A viable non-integer extension is crucial to the physical application of tetration, as the real world so rarely involves pure integers. To address this shortcoming, I am attempting to define an extension for non-integer heights that is consistent with basic identities and can be calculated using known functions. Using several relations I discovered, I was able to derive a full solution of non-integer tetration expressed in terms of what I call an infinite composition series. I was then able to use this formula to show that (e^(1/e))^^(-3/2) ≈ -1.295, which is consistent with the accepted value. Because this derivation was discovered incredibly recently, I have yet to fully test and validate the process, though I'm planning on coding an implementation to help me do so. Upon fully testing and validating my derivation, I plan to publish my complete extension for non-integer tetration. This may ultimately allow the operation to be fully integrated into the study of the natural world.

Awards Won:

American Mathematical Society: Third Award of $500

National Security Agency Research Directorate : First Place Award "Mathematics"