How Does Ring "Dance" in a Bowl?: Study on Chaos-Like Dynamic Behavior of Rigid Rings Using Lagrange Mechanical Analysis

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Even a simple ring’s motion, it is still difficult to get an analytical solution as the number of degrees of freedom is larger than four. In this project, I studied the interesting chaos-like dynamic behavior of a rigid ring moving inside a sphere in the frame of six degrees of freedom. First, by investigating the motion, I divided the general movement pattern into three different phases, summarized the motions of the ring with initial angular velocity, and studied the energy dissipation of each motion pattern. After that, I gave the general method of analyzing the behavior of rings in spherical constraint plane based on Lagrange mechanical analysis. By solving the Lagrange equations with Mathematica software, I was able to substitute various initial values and compare the results to experiments. Then, I conducted experiments to verify the theoretical model by a metal semi-sphere with a diameter of 25cm and three resinous rings with diameters of 4cm, 5cm and 5.5cm respectively. Finally, theoretical and experimental results were compared in the variation trends of six degrees of freedom over time and the trajectories of the ring obtained by plotting parameter diagrams or using Tracker software. It turns out that the theoretical results are highly consistent with the experimental data. The basic circular motion behavior is simulated using SOLIDWORKS software. The method used can be applied to solving complicated problems like non-in-plane motions, celestial motion problems with resistance and motion analysis based on non-Euclidean geometry.