Exact Analytical Solutions and Bifurcation Analysis of the Modified SIR Model With Time Dependent Population and Vaccination

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This study modified the Susceptible-Infected-Recovered (SIR) model, taking into account the population migration and vaccine to predict the dynamics of the third wave Covid-19 outbreak. The starting point is to locally stabilize the disease-free epidemic equilibrium point according to the eigenvalues. By performing the global stability using the Lyapunov method and Bendixon criterion, the research obtained the information that the disease-free equilibrium point is global asymptotically stable if the reproduction number fulfilled R0<1. If the behavior of a dynamical system changes suddenly as a parameter R0 is varied, then it is said to have undergone a bifurcation. At a point of bifurcation, stability may be gained or lost. Moreover, the study has proved that our system tends to the backward bifurcation condition. This condition reflects a condition of having endemic cases even though the parameter shows R0<1. Hence, the researcher derived the exact analytical solutions. The main idea is to convert the third-order nonlinear differential equation into the first-order integro-differential equation. To test the results, this study used the early third wave outbreak data in Jakarta, Indonesia. The comparison between the exact solution and the data shows a correlation of about R2 = 0 .9999. It also proved that the Covid-19 outbreak will be disappeared from the population if R0<Rc=-35,28. Eventually, this research shows that the vaccine can change the condition to the forward bifurcation. Using the numerical simulation, the higher speed of vaccination has produced the lower peak of infected cases. In conclusion, the increasing vaccination program is the best solution to solve the Covid-19 problem.