From the Manhattan Project to Statistics of Zeros of L-Functions

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The non-trivial zeros of L-functions encode important information about primes, which are essential to both number theory and modern cryptography. The zeros near the central point, s = 1/2, are especially important. For example, the Birch and Swinnerton-Dyer conjecture, one of the seven Clay Millennium Prize Problems meant to guide mathematics research for the next century, relates the order of the zero at the central point of the associated elliptic curve L-function with the size of the group of rational solutions. Moreover, the famous Generalized Riemann Hypothesis is a statement that these zeros all have real part equal to 1/2. The Katz-Sarnak Density Conjecture states that zeros of families of L-functions are well-modeled by eigenvalues of random matrix ensembles. For suitably restricted test functions, this correspondence yields upper bounds for the families' order of vanishing at the central point. We generalize results on the nth centered moment of the distribution of zeros to arbitrary test functions. On the computational side, we use our improved formulas to obtain significantly better bounds on the order of vanishing, setting records for the quality of the bounds. We also discover better test functions that further optimize our bounds. We see improvement as early as the 5th order, and our bounds improve rapidly as the rank grows (more than one order of magnitude better for rank 10 and more than four orders of magnitude for rank 50).

Awards Won:

Second Award of $2,000

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