Structural Dynamic Simulation: Mathematical Modeling of Bridge Deformations Using Analytical Methods

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Regarding transportation structures, simulation methods have been developed to simulate the deformational effects of vehicle travel over bridges for design verification, structural safety, and long-term damage and wear analysis. Over time, these foundational theories have evolved into many numerical methods for analysis, including the popular finite element method (FEM) to discretize partial differential equations that govern the structural dynamics. These simulation methods, though, often gather prohibitive computational costs and rely on truncation errors which jeopardize the integrity of simulation. This project sought to develop a direct analytical solution that can provides deformation simulation for transportation structures with greater accuracy and efficiency. Mathematical model was firstly derived based on classic simply-supported beam theories and first principles of force and moment, then extended to dynamic rolling contact for car-wheel contact. Direct analytical solution was then found via time frame separation, integration and geometric consideration. As verification for derivations, formulaic relationships were simulated with computer-generated constraints. Scatter plots of deformation metrics, fitted with FEM-calculated trend lines found R-squared values of >0.99, confirming the accuracy of the method. Furthermore, the dynamic simulation model was found to principal oscillations with little error (~0.15 normalized RMSE). While field-testing and further confirmation is warranted, by averaging a processing power of roughly 2 times faster than FEM, all the while upholding accuracy, the analytical model offers a promising step towards exploring direct analytical relationships as an alternative to numerical methods for the future structural simulations.