Modifying the ABCs of Number Theory

Sankaran, Akilan (School: Albuquerque Academy)

The abc-conjecture (Masser and Oesterlé) is an active problem that has remained open for several decades. The conjecture would imply a direct proof of Fermat's Last Theorem and also shine new light on the mystery of prime gaps. The statement of the conjecture concerns the asymptotic distribution of triples of integers (a, b, c), which satisfy: (1) a+b=c, and (2) a, b, c are divisible by few primes for their size (they are "round"). Such triples are measured using a quality metric; triples with highest quality are considered special. The standard quality, which is difficult to analyze, implicitly employs the concept of the Geometric Mean. I created several new classes of quality metrics that utilize the Doubly Geometric Mean (DGM) instead of the Geometric Mean. Through detailed investigation, I demonstrated that these metrics have qualitatively similar behavior to the standard quality, while privileging round triples. Furthermore, I connected Mersenne primes to high-quality triples. I also developed efficient algorithms to calculate large, high-quality triples, and implemented these algorithms in Python. My analysis allowed me to determine, with high efficiency, numerous triples with high qualities and millions of digits within a fraction of a second. These triples contribute new insights toward proving the abc-conjecture, which has applications in and outside of theoretical mathematics. I also formulated several conjectures about the new quality metrics, and illustrated the connection between these conjectures and various unsolved problems in mathematics, such as the twin prime conjecture and the Lenstra–Pomerance–Wagstaff conjecture on Mersenne primes.

Awards Won:
First Award of $5,000
American Mathematical Society: Certificate of Honorable Mention and One-Year Membership to AMS