

Gauss Circle Primes

Ehrenborg, Thomas (School: Henry Clay High School)

Given a circle of radius r centered at the origin, the Gauss Circle Problem concerns counting the number of lattice points $C(r)$ within this circle. It is known that as r grows large, the number of lattice points approaches πr^2 , that is, the area of the circle. This project seeks to study how often $C(r)$ will return a prime number of lattice points for r less than or equal to n . We call a value of $C(r)$ which is a prime number a Gauss Circle Prime. The researcher wrote a Java program to find the number of Gauss Circle Primes within a specified range of r . The Prime Number Theorem predicts that the number of primes less than or equal to n , called the prime number function $\pi(n)$, is asymptotic to $n/\log n$. We find that for n less than or equal to 2×10^6 : (1) the number $K(n)$ of Gauss Circle Primes for r less than or equal to n is also of order $n/\log n$, (2) $n/\log n < K(n) < \pi(n)$, and thus, (3) $K(n)$ gives a sharper approximation to $\pi(n)$ than the Prime Number Theorem. We include a heuristic argument that for all integers n the Gauss Circle Primes can be approximated by a constant times $n/\log n$. The experimental data implies this constant is 1.