Gauss Circle Primes

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Given a circle of radius r centered at the origin, the Gauss Circle Problem concerns counting the number of lattice points C(r) within this circle. It is known that as r grows large, the number of lattice points approaches pi r^2, that is, the area of the circle. This project seeks to study how often C(r) will return a prime number of lattice points for r less than or equal to n. We call a value of C(r) which is a prime number a Gauss Circle Prime. The researcher wrote a Java program to find the number of Gauss Circle Primes within a specified range of r. The Prime Number Theorem predicts that the number of primes less than or equal to n, called the prime number function pi(n), is asymptotic to n/log n. We find that for n less than or equal to 2 x 10^6: (1) the number K(n) of Gauss Circle Primes for r less than or equal to n is also of order n/log n, (2) n/log n < K(n) < pi(n), and thus, (3) K(n) gives a sharper approximation to pi(n) than the Prime Number Theorem. We include a heuristic argument that for all integers n the Gauss Circle Primes can be approximated by a constant times n/log n. The experimental data implies this constant is 1.