On Indefinite Reflection of Light Beam Off of Convex Decreasing Curves

Chiu, Da-Fu (School: Taipei City Non-school-based Experimental Education at Senior High School Level)

This study was motivated by a trance in my bathroom, looking at puddles on the ground, and imagining a curved mirror in mid-air. Then, emit a light beam to the curved mirror, in what circumstances can the photon keep reflecting forward indefinitely? The reflection process is characterized by a dynamical system to emit a light beam from $(0,0)$ with an initial angle ϕ_0. It then keeps reflecting between a curve $y = f(x)$ in the first quadrant and the positive x-axis. We first observe several geometric properties of the reflection process and derive the recurrence formula between adjacent incident angles. The formula can then be used to show that not all strictly decreasing curves $y = f(x)$ in the first quadrant lead to an indefinitely forward reflection, equivalently the light beam bounced back for some curves. By considering only positive $C1$ convex decreasing curves $y = f(x)$ with a limit of 1 as x approaches infinity, we can derive a sufficient condition on $y = f(x)$ for the dynamical system to be indefinitely reflective. In particular, we show that $f(x) = 1 + 1/x$ and all other $C1$ convex decreasing curves $y = g(x)$ whose limit approach to 1 such that $-1/x^2 < g'(x) < 0$ for $x \geq \cot(\pi/16)$ are indefinitely reflective. Examples include $g(x) = 1 + 1/e^x$ and $g(x) = 1 + 1/x^2$. We can strengthen the sufficient condition to hold for all $C1$ convex decreasing functions whose limit approaches ϵ where $\epsilon > 0$, but not the case when $\epsilon = 0$, such as the function $g(x) = 1/x$. Numerical data show that $g(x) = 1/x$ cannot reflect indefinitely for some small initial emitting angles. For $g(x) = 1 + 1/\ln(x)$, we conjecture that it is indefinitely reflective. Numerical experiment taken so far supports the conjecture but we hope to find a rigorous proof in the future.

Awards Won:

Third Award of $1,000