Extending the Vieta-Newton Theorem

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Systems of polynomial equations are of fundamental importance to mathematics, and have been applied to numerous areas, including physical problems, chemical reactions, robotics, communications, and computations of Nash equilibria in economics. (Sturmfels 2002). For an important type of system of polynomial equations, the Vieta-Newton Theorem provides a complete and elegant solution: solving for a set of variables given their symmetric power sums is equivalent to solving for the roots of a univariate polynomial equation. This result has been extended in recent studies to a few new yet still restrictive situations. The goal of this paper is to extend the Vieta-Newton Theorem to its most general case, where all variables can have arbitrary coefficients. We show that solving such a system is equivalent to solving a smaller system of polynomial equations in fewer variables and using the result to solve one or two univariate polynomial equation(s) of lesser degree(s) whose roots are the remaining variables. The results contribute to the general theory of systems of polynomial equations by providing new tools for studying solutions theoretically, as well as by providing algorithms for computing such solutions numerically. Our theoretical results can be directly applied to enhance the capabilities of computer algebra systems such as Mathematica and Maple. The results also have great potential for applications in error correction of Reed-Solomon codes (widely used in DVD, QR codes, bar codes, cell phones, satellite communications, etc) with more arbitrary errors, where the type of systems studied here arises naturally (Wu and Hadjicostis (2004)).

Awards Won:

American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)

National Security Agency Research Directorate : Second Place Award “Mathematics”

American Mathematical Society: Second Award of $1,000