

# Solving High-Degree Equations Using Modern Technology

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Math helps us in solving physical, biological, chemical and everyday problems which require the use of high degree polynomial equations. Unfortunately, solving these types of equations is not a part of the school program. In school, students only learn how to solve quadratic equations with the discriminant. Out of the high degree polynomial equations we only study specific 4th degree equations called biquadratic equations with the form:  $ax^4 + bx^2 + c = 0$ , where we change the variable with  $z$ , where  $z=x^2$ . This project aims to create an innovative iterative algorithm (using C++) to solve polynomials with rational roots (not those with roots of irrational or imaginary numbers). Bezout's theorem states that when dividing  $P(x)$  polynomial by  $(x-a)$  we will get a remainder of  $R$ .  $P(x) = (x-a)g(x) + R$ . Result 1:  $x=a$  is a solution to  $P(x)$  only and only if,  $P(x)$  gets divided by  $x-a$  binomial without a remainder. Result 2: If  $a_n=1$  and the polynomial coefficients are whole numbers, and constant term  $a_0$  is divided by the solution, then, every rational solution is a whole number. Let's discuss this  $n$ th degree equation:  $a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0 = 0$  where  $x$  is the variable and  $a_n, \dots, a_0$  are whole numbers. Let's say a rational number  $x=p/q$  is a solution, which is an irreducible fraction, where:  $q$  is divisor of  $a_n$ , and  $p$  is the divisor of  $a_0$ . In this project we will apply our algorithm to solve several representative equations, such as:  $x^4 - 7x^3 + 7x^2 + 35x - 60 = 0$ . We write the set of divisions of constant term  $a_0 = -60$ . Instead of  $x$  we put the elements from  $M$ . If it satisfies the equation, we divide the polynomial by  $(x-x_1)$  binomial.  $(x^4 - 7x^3 + 7x^2 + 35x - 60) : (x-3) = x^3 - 4x^2 - 5x + 20$ . We reduce the degree of the equation by 1 and we repeat this process until we get a quadratic equation.