Further Understanding of Gyroscopic Precession: Analysis in the Rotating Frame

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Despite precession being a centuries-old problem, it remains a topic worth delving into. The well-known quadratic equation that yields the rate of precession is mathematically convincing, yet finding its intuitive or physical meaning is challenging. Moreover, the conventional discussion on the stability of gyroscopic precession is imperfect, since the effective potential method presumes there are no external fluctuations other than those in the direction of the coordinate of interest. Surprisingly, a slight shift in perspective can simultaneously address both problems mentioned above. In this study, we suggest analyzing the precession in a frame of reference precessing with the gyroscope. The first problem we encounter involves deriving the quadratic equation in the rotating frame. We significantly simplify the calculation of fictitious torques using a tensor substitution devised in this study. This allows us to obtain a more informative form of the equation from torque equilibrium. The new form of quadratic equation, with a slightly altered definition of angular velocity due to the change of frame, naturally reveals the physical meanings of the terms. Afterward, the missing possibilities in the effective potential method are addressed with a novel stability test. Our test, an improved eigenvalue test based on Jordan–Chevalley decomposition, can deal with systems whose Jacobian of Hamiltonian flow is not diagonalizable. The marginal stability of the precessing gyroscope in the azimuthal direction has been validated with this test. Ultimately, the new point of view not only deepens the understanding of gyroscopic precession but also offers a versatile stability test applicable to both physical and nonphysical systems that can be described as Hamiltonian.