

A Study on Arc Index of Theta Curves

Lee, Yoonsang (School: Korea Science Academy of KAIST)

This study explored various general properties related to the theta curve's arc index. The arc index not only serves to classify spatial graphs but also plays a crucial role in computing a significant topological invariant called floer homology for knots. Therefore, the investigation of the theta curve's arc index is expected to greatly benefit future research related to theta curves. I first derived two lower bounds for the theta curve's arc index using the arc index of constituent knots, and also determined a lower bound in terms of crossing number by using a loop-link diagram, one of the representations of the arc presentation. Furthermore, by identifying the relationship between the Yamada polynomial and the number of caps and crossings in stacked tangles, I obtained a lower bound in terms of the highest and lowest degrees of the Yamada polynomial. This addresses the shortcomings of non-sharp bounds for cases such as Brunian theta curve. For the upper bound, I derived by proving the existence of an arc presentation with the number of half-planes equal to the crossing number plus 4. The spoking algorithm and a graph-theoretical lemma were used to justify its existence. Using the obtained bounds and the binding circle method, I determined the ranges of the arc index of pretzel theta curve and Kinoshita-Wolcott theta curve. Additionally, for all theta curves with crossing numbers of 7 or less, I computed the values of each bound, confirming that the newly derived bounds in the study are tight and thus meaningful.