Euler-phi Partitions, p-Euler-phi Partitions, Inverse-p-Euler-phi Partitions and Their Generating Functions

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Partitions of integers n are different representations of n as a sum of positive integers. They have applications in a variety of maths and physics topics including distribution problems and statistical mechanics. In this project, Euler-phi partitions, p-Euler-phi partitions and Inverse-p-Euler-phi partitions are introduced as new partition definitions. Let n and p positive integers. Different representations of the number n as the sum of positive integers that are relatively prime to p are defined to be p-Euler-phi partitions of n. For p=n, these partitions are also called Euler-phi partitions. Different representations of the number n as the sum of positive integers that are not relatively prime to p are defined to be Inverse-p-Euler-phi partitions of n. In order to determine these partitions and compute their counts for relatively large numbers, several Python codes were written. These were used to check our insights about theorems before proving them. Additionally, over 40 generating functions were obtained for the partitions that were introduced in this project. These were then used to obtain asymptotic formulas which can be used to determine the leading term and pave the path for further studies. Furthermore, numerous theorems for particular cases of p and positive integer n were proved. Some of these are as follows: * When p is a power of an odd prime number, exact formulas for the number of p-Euler-phi partitions of n into 2 or 3 parts were obtained. * When p is a prime number, it has been shown that the number of Inverse-p-Euler-phi partitions of np is equal to the number of partitions of n. Moreover, a theorem that connects p-Euler-phi partitions and Inverse-p-Euler-phi partitions was proved by generating functions and Cauchy product.