

Integration of Sequences

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In my research project I want to generalise the well-known concept of Riemann integration of functions on sequences in order to obtain a new tool for the investigation of sequences. To do this, I define an integral for sequences based on the Riemann integral and then investigate which sequences are integrable. I prove necessary and sufficient conditions for sequence integrability. It turns out that integrability for a sequence is a stronger condition than Cesàro-convergence: Every integrable sequence is Cesàro-convergent, but not every Cesàro-convergent sequence is also integrable. As a sufficient criterion for integrability, we find a condition that resembles Lebesgue integration: If suitable preimages under the sequence are "measurable", meaning that we can assign them a kind of length, the sequence is integrable. For sequences with finitely many accumulation points, we prove that this criterion is also necessary.