On an Approximation of Divisor Sum Functions With Bernoulli Polynomials and the Hardy Littlewood Function

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Is a Bessel representation of exponential sums corresponding to divisors obtainable, and if so, viable? In an original expression I derived relating the divisor functions at individual points and their averages by exploiting the periodicity of roots of unity, in a sense inverting it, there is an exponential sum called the Hardy Littlewood function. Moreover, Voronoi uses a generating function transform using complex analytical tools to describe the error between the average of the divisor function and its asymptotic main terms. With both in mind, I similarly used a complex analytical generating function transform. To get an exact representation of the sum, I changed the contour from a circle to a keyhole. The result of this procedure is a conversion between any exponential sum and a Hankel transform. To fully express the exponential sum in terms of a Hankel Transform, a subset of the geometric series that's related to the amplitude function of the exponential sum must be studied. Also, I expressed the Hankel Transform in terms of the FHA cycle, and showed a correlation between an oscillatory integral and exponential sum: similar objects with different measures. The literature shows that both of these can be studied: with methods from Mackinnon's paper on asymptotic expansions of Hankel Transforms and the saddle point method. These results open up a new gap for sums involving subsets of the geometric series by creating context for them, and different methods besides the Van Der Corput processes A and B are discovered.