

# Along the Path of the Great Kaprekar: A-Function, Repunits and Their Properties

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Inspired by the famous function of the D.Kaprekar  $K(n)=n+S(n)$ , I invented and investigated various properties of a new function  $A(n)=(n-S(n))/9$  where  $S(n)$  is the sum of digits of the natural number  $n$  in decimal notation. The values of the function  $A(n)$  are called a-generated numbers and for the given value  $m=A(n)$  the argument  $n$  is called its a-generator. If a number  $p$  has no a-generators, then  $p$  is called an a-self number. The rank  $r(n)=u$  of a number  $n$  is the smallest number  $u$  such that  $A(A(\dots A(n)))=A^{(u)}(n)=0$ . Repunits are numbers whose decimal notation consists of only units. The connection between  $A(n)$  and repunits is established. Two new formulas for the class of repunit numbers are proved. Non-decreasing of the function  $A(n)$  and its other properties are proved. It is established that every a-generated number has exactly 10 a-generators. The infinity of a-self numbers is proved. It is shown, that fraction of a-self numbers within all numbers with given number of digits is one tenth. Formulas for "neighboring" (sequence of consecutive) a-self numbers with sequence lengths 1 and 2 are found. A pattern and a hypothesis for the distribution of "neighboring" a-self numbers with any other sequence lengths are formulated. The equations  $K(n)=q \times A(n)$  and  $A(qn)=q \times A(n)$  are completely solved. It is proved that the function  $r(n)$  is non-decreasing.