Measuring Quantum Entanglement Entropy in Gaussian Boson Sampling

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Quantum computing has significant potential to generate unparalleled innovations in lattice modeling, encryption schemes, protein folding, prime factorization, and countless other fields of scientific research. I rigorously derived two formulas for the average quantum entanglement in Gaussian boson sampling, a common real-world experiment performed by quantum computers, using both computational and analytical techniques. I began with a non-closed form of the Rényi entropy, a quantity that measures entanglement in a quantum system, and worked towards a closed form by expanding matrix terms, using Taylor series, defining helper matrices, and utilizing the binomial theorem for negative exponents. Previous efforts have only quantified the entanglement to one type of entropy, but my formulas extended the entanglement to an infinite class of entropies. Next, I analyzed two limiting cases of my formulas for very low- and high-energy photons, and discovered that the high-energy case yields a truly random quantum substate. Additionally, I investigated a behavior of entanglement called the typicality, which can be generalized to other bosonic systems, by showing that the variance of the entropy is constant. Finally, to experimentally validate my theoretical findings, I constructed a Python simulation that calculated the entropy in Gaussian boson sampling. The simulation provided conclusive support for my two formulas, and also featured small errors for specific input values. Overall, my research puts a firm mathematical basis to examining how entanglement affects the quantum computational speed of Gaussian boson sampling, accelerating the progress towards feasible real-world quantum computing and the subsequent innovations across scientific research.