Some Properties of the Jacobi Point in the Triangle

Orlova, Daria (School: Kangwon Science High School)

Given a triangle and a set of three angles, celebrated geometrical theorem of Jacobi (1825) produces a new triangle and a point (Jacobi point) in perspective with the first. Different properties of Jacobi triangle were investigated by G.Vickers in a paper "Reciprocal Jacobi Triangles and McCay cubic"(Forum Geometricorum, v.15 (2015), p.179-183).The purpose of our work is to study some interesting properties of Jacobi point. Namely we find necessary and sufficient conditions of coincidence of Jacobi point with one of the following triangle centres: the centre of incircle, the centroid, the circumcentre, the orthocentre. All conditions are stated on trigonometrical language and express the equality of the ratio of sine of some angles. We recieved interesting corollaries of the coinciding conditions for regular triangles and constructed nontrivial examples of nonregular triangles in which Jacobi point coincides with one of triangle centres. Our proofs are based on Ceva's theorem, different properties of bisectors, medians and heights of a triangle. We use also algebraic method such as the sine rule and trigonometric transformations.