## Polynomials in Z[x] and Irrationality Measure

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The field of Diophantine approximation is a branch of number theory which concerns itself with certain properties of real numbers in relation to the rational numbers. In specific, one may define the irrationality measure of a real number to give a meaning to how "close" it is to the surrounding rationals. Several important theorems have been proved recently, including the determination of the irrationality measure of all algebraic numbers. I hypothesized that I would be able to prove a relationship between the irrationality measure of a number $x$, and the irrationality measure of a polynomial $P(x)$. In addition, I wanted to find out if I could do so without the use of calculus or analysis, as many concise proofs of theorems in the field of Diophantine approximation rely on deep theorems from these areas. I present a novel theorem which relates the irrationality measure of a real number $x$ to the irrationality measure of the real number $P(x)$, whenever $P$ is a polynomial with rational zeros. The proof shows that the irrationality measure of $P(x)$ is necessarily bounded below by a function of the irrationality measure of $x$. I prove this relation using algebraic methods and induction. I then extend this proof to complex numbers. This is accomplished by replacing the real absolute value with the complex norm in the proof. I show that the same relation holds, wherein the polynomial P has roots which have rational real and imaginary parts.

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