

# An Addition to the Problem of Points

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The problem of points is a historical problem confronted by many mathematicians throughout history. It presents an interrupted series of games and asks the question of how the probabilities of victory for each player can be found. Through the course of this research, a variation of this problem that presented a series of games in the format of  $(X, Y)$  best of  $z$  was considered. In this format  $X$  and  $Y$  are both players,  $z$  is the total amount of games and  $n$  is the amount of remaining games. A preexisting method of obtaining the probabilities, which used binomial coefficients to define the amount of ways for a player to win a series over the total amount of possibilities for outcomes, was expanded using the relationships between  $x$ ,  $y$ ,  $z$  and  $n$ . An early demonstration of the probabilities was first obtained under the assumption that  $|X - Y| = 1$  which branched off into cases for even  $n$  and odd  $n$  since ties would only be possible in the latter of these cases. A complete demonstration was later found that applied regardless of the value of  $|X - Y|$ . This demonstration also branched outwards into two cases due to the possibility of ties. Afterwards, the more complete probabilities were shown to be equal to the incomplete probabilities when  $X + 1 = Y$ , thus showing the equivalencies between both demonstrations. These definitions were then applied to visually representing an example series progression on Pascal's Triangle. In this representation every obtainment of an individual game was a move upwards in row.