Discrete Ricci Flow on Discrete 3-Manifolds

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In this project, Ricci flow and normalized Ricci flow were studied in a discrete 3-dimensional setting, where the manifold of interest is the boundary of a 4-simplex. This manifold is the discrete analogue of a 3-sphere. The solutions to the discrete Ricci flow equation (derived by a first-order Taylor approximation of the discrete Ricci curvature) were shown to have all associated properties of the solutions to the smooth Ricci flow equation; solutions that correspond both to shrinking the manifold to a point and to exhibiting curvature smoothing were observed. Curvature smoothing is the key property that deforms a manifold to a sphere, which led to the proof of the Poincaré conjecture. Unexpectedly, in the discrete setting, this smoothing behavior only occurs when the flow is solved in negative time, as demonstrated by a calculation on the approximate solution to the discrete Ricci flow equation. Three different normalizations (preventing the manifold from shrinking to a point in finite time) of the discrete Ricci flow were also examined, with the conclusion that these equations must also be run in negative time to have the suitable properties of the solutions to the smooth Ricci flow equation. In particular, the solutions will converge to the manifold with constant Ricci curvature. As Ricci flow can be applied to study the curvature of space-time under gravity, my results may be helpful in studying a discrete approximation of space-time.

Awards Won:

Second Award of $2,000