

# A 6-Chromatic Unit Distance Graph in Space

Li, Stephanie

Lee, Thomas

The chromatic number of the  $n$ -dimensional Euclidean space is the minimum number of colors with which the entire space can be colored so that every two points one unit apart have different colors. A unit-distance graph in  $n$  dimensions is a graph whose vertices are points in the  $n$ -dimensional space and whose edges are pairs of points unit-distance apart. Clearly, the chromatic number of an  $n$ -dimensional space is at least as large as the chromatic number of some finite unit-distance graph. The chromatic number is greater than or equal to  $n+1$ , since every two vertices of an  $n$ -dimensional regular simplex of unit side-length require different colors. The spindle construction, ascribed to Moser and Moser for  $n = 2$  and Raikii for  $n > 3$ , uses regular simplices to demonstrate that the chromatic number is greater than or equal to  $n + 2$ . In 2002, Nechushtan proved that the chromatic number in 3 dimensions is greater than or equal to 6 and claimed that his proof can be used to construct a unit-distance graph which requires six colors and has no more than 400 vertices. By rotating and transforming Nechushtan's basic unit-distance graph with chromatic number 5, we construct an explicit unit-distance graph in 3 dimensions with chromatic number 6 that has only 79 vertices and 250 edges. Our methods can be applied to certain 2-deficient,  $x$ -chromatic unit-distance graphs in order to construct graphs with chromatic number  $x+ 1$ .

## Awards Won:

Fourth Award of \$500