

# n-Dimensional Fractions and a Generalized Calkin-Wilf Tree

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This paper provides a generalization of the Calkin-Wilf tree. The Calkin-Wilf tree is an infinite binary tree in which all nodes are labeled by reduced positive rationals and each reduced positive rational labels exactly one node. An n-dimensional fraction over  $\mathbb{Z}$  is a formal symbol  $a_1/a_2/\dots/a_n/b$  where  $a_i, 0 \neq b \in \mathbb{Z}$ , is in reduced form if and only if  $\gcd(a_1, a_2, \dots, a_n, b) = 1$ , and is positive if and only if  $a_i, b > 0$ . The purpose of this research project is to construct and investigate the graph structure of the n-dimensional Calkin-Wilf tree which has the following property: all nodes are labeled by reduced positive n-dimensional fractions and each reduced positive n-dimensional fraction labels exactly one node. The n-dimensional Calkin-Wilf tree is constructed by the reverse Euclidean algorithm. The graph structure of the n-dimensional Calkin-Wilf tree is determined completely by  $\text{Diagram}(n+1)$  which is a finite directed graph in which all nodes are labeled by partitions of  $n+1$ , each partition of  $n+1$  labels exactly one node, and each edge is labeled by a certain positive integer.