Developing an Algorithm to Simulate and Visualize Abelian Sandpiles on Three Types of Lattices

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Abelian Sandpiles have numerous contributions in the fields of Mathematics, Computer Science and Physics. As they are an example of systems displaying Self-Organized Criticality, Cellular Automaton and exhibit abelian properties studied in Group theory. A number of algorithms were developed to visualize and simulate sandpiles on certain types of lattices for this reason. However, the visualization and simulation of sandpiles was mostly limited to Cartesian grids (the square lattice). In this research, the approach was to develop an algorithm to visualize and simulate sandpiles on three types of lattices, of which are two that were not previously used for their simulation and visualization, using the programming language Python. The implementation of the algorithm was designed to simulate the process of firing chips and visualize sandpiles using an initial configuration named “Toppling” and mainly consisted of three functions. To confirm the correspondence between the properties of sandpiles simulated on the two proposed types of lattices and sandpiles simulated on Cartesian grids, statistics of avalanche sizes and their frequencies were collected and confirmed to form an inversely proportional relationship. The data was modeled on a logarithmic scale and a line of best-fit was graphed with the data. The slope of the line was calculated and confirmed to approximately equal -1. This indicates that sandpiles modeled on the two proposed lattices display similar behavior to those on Cartesian grids. Finally, this algorithm could be used as a readily-available method for the analysis of the structure of Abelian Sandpiles and study of their properties.