A Geometrical Analysis of Harmony and Voice Leading in Music Using Quotient Orbifolds

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Western music is based on two concepts – Harmony, which is the simultaneous sounding of multiple notes (chords), and Voice Leading, which is the temporal motion of these notes (voices). The incorporation of these two concepts is called counterpoint. A chord can be defined as an n-tuple in an n-dimensional non-Euclidean geometric space called an orbifold. The voice leading from one chord to another within the orbifold is defined by vectors in the space. In this study, orbifolds are used in an innovative way to analyze musical harmony and voice leading to achieve two goals. First, it is shown that orbifold analysis is a powerful geometrical technique to study composed music. In particular, the evolution of counterpoint from 9th to 18th centuries is studied using orbifold mapping for the first time and the visual orbifold analysis is shown to be a powerful tool to glean new insights into the compositions. Second, a new technique using orbifold analysis has been developed to identify good sounding music and the limits of playability as a guide for efficiently composing music. Over 300,000 chord transitions were analyzed with a program and a random sample was graphed onto the orbifold. Some interesting geometric properties of voice leadings were revealed visually for the first time and this innovative visual technique identified the ones that violated counterpoint rules or were unplayable. In conclusion, the study demonstrates that a geometrical analysis of harmony and voice leading is invaluable in gaining new insights into compositions, something of immense value to music theorists and composers. The study also developed a powerful new technique using orbifolds to efficiently compose new music, and this should prove extremely useful to musicians and composers.

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