Triangular Circle in a Square

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Figures of constant width are figures able to rotate inside a square being tangent to all its sides simultaneously. The Reuleaux triangle is the most famous of these figures, being the base of claw mechanisms, rotary Wankel engines and even Watts drills allowing boring of square holes. However, such holes differ from the square shape by round corners. I built a model of such a drill and found out that some internal points of the triangle move along the trajectories with acute angles looking more like a square compared to boundary points. I explored a one-parameter family of trajectories generated by moving points of Reuleaux triangle at its rolling along the square. Parameter t describes point position along the symmetry axis inside or outside the triangle. There are 5 bifurcation points showing different behavior of trajectories. Outside the edge points t1 and t5 the trajectory is a smooth closed curve without selfcrossings. Values of t in two next semi-intervals give us a closed curve with 4 points of self-crossing. At t=t4 and t=t2 the trajectory consists of 4 closed curves crossing in 1 point. In two inner intervals the loops of the closed curves are superimposed. In central point t3=0 the trajectory of the triangle center consists of 4 elliptical arcs. The ratio of the area limited by the closed curve and the area of the circumscribed square was calculated for all the values. All calculations are precise and presented in irrational numbers, they do not contain approximate computations. The program was designed in FlashDevelop environment to illustrate the mathematical proof. Placing an additional arrangement in bifurcation point t1 one can improve Watts drilling process reaching the angles with the accuracy up to 0.07% that is up to 30 times closer to the square.

Awards Won:

American Mathematical Society: hfghfghf