The Secrets in the Coin Toss: Fairness Evaluation in the Binary Decision Based on Dynamics

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Tossing a coin is often used to make a binary decision. For the coins in daily life, there exist protuberant parts, portrait or decorative pattern on the surface. The mass is non-uniform distributed. In this study, we investigate the dynamics of uniform and non-uniform coins, and compare their fairness in the coin toss games. The motion of the coin in the air can be modeled with a dynamical system. It depends deterministically on the initial configuration. Firstly, we introduce a reference frame and body-fixed frame to describe the coin's orientation. Secondly, we establish the differential equations and obtain the trajectory of the normal vector of the coin. Thirdly, we provide the probability of heads under specific initial conditions. Lastly, we investigate the empirical distribution of the initial parameters by high speed filmed real flip experiments. In this work, we reveal the different dynamic behaviors for non-uniform and uniform coins. For uniform coins, the normal vector traces out a circle. For non-uniform coin, the normal vector sweeps out a ring. We also provide the criterion of heads up and the formula for calculating the probability of heads. The well-known results from J. B. Keller and P. Diaconis are the special cases. For the first time, we evaluate the fairness in tossing non-uniform coins, and find that non-uniform coin is fairer than uniform coin for a wide range of initial conditions.

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