The Algebra and Geometry of Quasicategories

Devalapurkar, Sanath

The inherent stability in algebraic K-theory proved in my previous paper suggests a general notion of stability in higher category theory. In this paper, I use the abstract notion of a scaled simplicial set to provide a definition of a stable quasi-2-category, an example of which is the quasi-2-category of stable quasicategories. We establish the consistency of this definition. We also introduce the notion of an 2-fold complete Segal operad by generalizing Barwick's definition of complete Segal operads to the notion of a 2-fold complete Segal operad. Using this, we study the notion of a 2-ring, which generalizes the notion of a ring spectrum, and, more importantly, sets a new foundation for a generalization of commutative algebra. Ordinarily, ring spectra fit together into a very complicated category called the stable homotopy category, which is the main object of study in (stable) homotopy theory. We provide multiple ways of extracting incredibly important information about the stable homotopy category from stable quasi-2-categories by defining and studying a "derived" analogue of the moduli stack of formal groups. Many interesting results about the basic structure of this derived moduli stack are proved, which in turn leads to a lot of interesting venues for further research, including a possible definition of n-rings for n>2.

Awards Won: