

# The Investigation of an Impartial Normal Play Game

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This project investigated the winning values of the following Nim variant: There exists set A and set B, with some number of elements in each. On a player's turn, the player may reduce set A by  $x$ , set B by  $y$ , or both sets by  $z$ , where  $x$ ,  $y$ , and  $z$  are positive integers. The player who takes the last turn, or reduces A and B to zero, wins. "Safe spots" are winning values of sets A and B, such that the player who reduces the sets to the safe spot values will win. By analyzing the relationships between safe spots, a sequence was derived to represent the difference between adjacent safe spots. Subsequently, a recursive equation was developed to represent the sequence. The average distance between safe spots was calculated and found to be equal to  $\phi$ , where  $\phi$  is equal to  $f_n/f_{n-1}$  as  $n$  goes to infinity, where  $f_n$  is the  $n$ th Fibonacci number. Because of the correlation to the Fibonacci series, the project results may have applications in biology, number theory, geometry, and other fields of math and science.