Cosheaf Theoretical Constructions in Networks and Persistent Homology

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Persistent homology has recently emerged as a powerful technique in topological data analysis for analyzing the emergence and disappearance of topological features throughout a filtered space, shown via persistence diagrams. Additionally, (co)sheaves have proven to be powerful instruments in tracking locally defined data across global systems, resulting in innovative applications to network science. The goal of this project is to combine the topological results of persistent homology and the quantitative data tracking capabilities of cosheaf theory to develop novel techniques in network data flow analysis. Specifically, we use cosheaf theory to construct persistent homology in a framework geared towards assessing data flow stability in hierarchical recurrent networks (HRNs). We use cosheaves to link topological information about a filtered network encoded in persistence diagrams with data associated locally to the network. From this construction, we use the homology of cosheaves as a framework to study the notion of “persistent data flow errors.” That is, we generalize aspects of persistent homology to analyze the lifetime of local data flow malfunctions. We proceed with several constructions reminiscent of the persistent homology of filtered topological spaces to fit our network theoretical environment. We conclude with an algorithmic construction of persistence diagrams that visually parameterize network data flow errors, thus enabling novel applications of statistical methods to study data flow malfunctions. Our results can be applied to analyze data flows in complex systems such as financial, social, and biological networks.

Awards Won:
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