A Generalisation of the Determinant to Rectangular Matrices: Implications in Gauge Theory

Pallavi Sudhir, Abhimanyu

The determinant is an operator on square matrices, such that the determinant of any square matrix yields a scalar. There exist a few generalisations to non-square matrices, such as the geometric determinant and the vector determinant. The determinant-like function is a class of such generalisations which satisfy the property of determinant-wedge equivalence. In this project, I demonstrate that the vector determinant is a determinant-like function, i.e. it satisfies the property of determinant-wedge equivalence. I then proceed to find a few properties of the vector determinant/determinant-like function. The first of these properties relates the determinant-like function to solutions for systems of m linear equations with n variables. The second property defines a new class of groups called rectangular groups containing rectangular matrices whose determinant-like function satisfy specific conditions. Finally, the implications of the determinant-like function in gauge theories in Physics are discussed. It appears that a gauge theory based on a rectangular group results in gauge forces that are the curvature of bundles with a rectangular metric tensor. A classical approach is applied using General Relativity and Differential Geometry to study the dimensions and properties of various tensors associated with a manifold with a 4 by 3 metric tensor.

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