Characterizing the Constructible N-Division Points of the Rational C-Hypocycloids through Straightedge and Compass Constructions

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One of the most famous mathematical problems concerns constructing figures with the aid of an unmarked straightedge and compass, specifically constructing regular polygons. Such construction problems concerning regular n-gons can be generalized to consider the n-division points of any closed curve C (n points dividing C into pieces of equal arc length). This year, I worked towards three major goals in the field of n-division point constructions: understanding the constructible n-division points of hypocycloids, characterizing the constructible n-division points of an entire family of curves for the first time in this field, and resolving a longstanding unaddressed disparity concerning the presence (or lack thereof) of a closed curve C when considering the constructible n-division points of C. I used Galois theory, abstract algebra, and algebraic number theory in order to characterize constructible numbers and prove two theorems. I showed that the n-division points of all rational c-hypocycloids are constructible with an unmarked straightedge and compass for all integers n, given a pre-drawn hypocycloid. I also considered the question of constructibility of n-division points of hypocycloids without a pre-drawn hypocycloid in the case of a tricuspoid, concluding that the n-division points of a tricuspoid are only constructible in this manner when n | 6 (the n-division points are always constructible when given a drawn tricuspoid).

Awards Won:

American Mathematical Society: First Award of $600

European Organization for Nuclear Research-CERN: Third Award $500