Solution to the Realization Problem for Two Element Delta Sets

Colton, Stefan

The study of error correcting codes seeks to minimize the redundancy needed for the accurate transmission of data on a fuzzy channel. Interference necessitates costly amounts of redundancy, reducing the channel capacity of systems affecting billions. Weierstrass points on algebraic curves have been key in developing more efficient algorithms, and each Weierstrass point has an associated numerical semigroup. A numerical semigroup is a set with closure under addition, the zero element, and a finite complement of the natural numbers. The delta set of a numerical semigroup is a way of measuring the complexity of its factorizations and thus important in understanding Weierstrass semigroups. It is largely unknown which delta sets exist, and while it is straightforward to calculate the delta set of a numerical semigroup, it is extremely difficult find a numerical semigroup with a particular delta set. This study first considered the abnormal {1,3} delta set. An algorithm for generating a numerical semigroup with any 2 element delta set was discovered and then proven. Additional formulas were conjectured for an even more expansive variety of delta sets; also discovered was a family of complete intersection numerical semigroups, which have been studied in relation to error correcting codes. This work improves our understanding of numerical semigroups, which can lead to more efficient codes. This is critical for any system where noise interferes with data, such as telecommunications and deep space communications. Telecommunications alone is a 4.7 trillion dollar industry; even small increases in efficiency could leave billions to benefit society.

Awards Won:
American Mathematical Society: Second Award of $200