Rigorous Constraints on Exchange for the 3-Body Problem

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Exchange in the three body problem occurs when one body displaces a previously bound member of a binary system. Exact conditions for when such an exchange can occur are established through a Theorem that utilizes the amended potential function of the general 3-body problem. Such barriers to motion have been studied previously; however, this paper establishes these conditions in general through analysis of the topological properties of the amended potential function. Specifically, the analysis uses the Morse indices of the amended potential's stationary points to prove the result. Stationary points exist when the three bodies are arranged in a line with appropriate spacing (a.k.a. Euler configurations) and when they are in an equilateral triangle (a.k.a. Lagrange configurations). One of the basic tenets of Morse theory is that a function only changes its topological type at stationary points, so the function's connectedness is studied for levels of energy "in between" the levels at which stationary points exist. The analysis reveals that for energy levels less than the least stationary value, no exchange can occur. As the energy increases, the barriers to exchange disappear precisely at each Euler level, until all exchange is permitted. The Lagrange energy level does not affect barriers to exchange.

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