On the Constructibility of n-Division Points of Certain Polar Curves by Area

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In this paper, we develop a method for a problem not heavily touched upon by existing mathematical literature: evaluating the constructibility of n-division points by area that can be applied to any given closed polar curve, with n-divisions defined as rays starting at the origin that divide a closed polar curve into n sectors of equal area. Positive results were found for the ellipse, the inner loop of the Maclaurin trisectrix, and the k-petaled lemniscate. Our method involved finding a closed form solution that expressed the radius of the curve in terms of the traversed area in order to analyze whether certain divisions were constructibly possible. When this expression was algebraic and constructible, using this method yielded the following results: the n-divisions of an ellipse are constructible if n is constructive for a circle, and the n-division points of the inner loop of the Maclaurin trisectrix and of the k-petaled lemniscate are constructible for all positive integers n. Though the results for the ellipse have already been proven by Kepler, the results for the Maclaurin trisectrix and the k-petaled lemniscate are original. Furthermore, our method suggests that other curves with transcendental expressions for radius in terms of area do not have constructible n-division points. A secondary result of our work is that the constructible n-division points are also constructible without the given curve for the ellipse and the inner loop of the Maclaurin trisectrix, though not for most k-lemniscates.