Fuzzy Structures with Application to Differential Topology, Manifold Learning, and Specialized Concepts in Mathematics

Tramontano, Jared

The main purpose of the project is to introduce a notion of Differential Topology in the setting of Fuzzy Set Theory, introduced by Zadeh in the 1960's. As Fuzzy Set Theory is just a generalization of set theory, Fuzzy Differential Topology is just a generalization in the normal Euclidean setting. All concepts were developed with application to Manifold Learning in mind. Namely, one may consider fuzzy data manifolds in computer science. The standard setting of Differential Topology was extended to the notion of Fuzzy Topological Vector Spaces. Although slight attempts have been made previously, such attempts lacked rigor and depth. The new notions developed include higher dimensional fuzzy sets, Fuzzy Topological Separation Axioms, Fuzzy Differentiation, Fuzzy Atlases, Fuzzy Tangent Bundles, Fuzzy Cotangent Bundles, and Fuzzy Lebesgue Integration. These notions give rise to a proper foundation for Fuzzy Differential Topology, where most concepts, such as homology, should have suitable analogies in these spaces. The notions I have developed could lead to significant future work, such as analogies of the prominent theorems of Differential Topology; the Whitney Embedding Theorem, for example.