A New Method to Determine the Properties of Solutions to One-Variable Cubic Equations with Integral Coefficients

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Purpose of the research: My thesis analyzes the properties of real solutions to cubic equations with integral coefficients. Using knowledge from basic number theories, calculus, and the method of symbolic-graphic combination, the method in my thesis can efficiently identify the number of rational and irrational roots in such equations without much calculation. Procedure/order of reasoning: Unlike the conventional Cardano method imposing variable transformation to obtain formulas, the method in my thesis follows a different order of reasoning. The steps are described below: 1. Establish supportive theorems concerning the rational roots of the equations. 2. Use the conclusions from step 1 to analyze the conditions for irrational roots, and introduce more complex theorems for subsequent discussion. 3. Integrate the theorems in step 1 and 2 for central theorems 3.1-3.3, which state conditions for such cubic equations to have 1,2 and 3 irrational roots. 4. Examine my discussion of the problem, and find possible directions of new researches. Starting with a theorem about number theory common in Olympiad-level mathematics courses, my thesis introduces 3 theorems about the irrational roots of integral-coefficient cubic equations. These theorems form an efficient method to determine the characteristics of real solutions to the equation. Conclusion: All cubic polynomials have at least one real zero. When the coefficients of a cubic polynomial's terms satisfy different sets of conditions, the cubic polynomial has different types of real zeros. Depending on the coefficients of different terms, a cubic polynomial can have one, two or three irrational zeros.