Weighted Catalan Numbers and Their Divisibility Properties

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The weighted Catalan numbers, like the Catalan numbers, enumerate various mathematical objects. For example, the number of Morse links with n critical points is the n-th weighted Catalan number, L_n, with weights 1^2, 3^2, 5^2,..., (2k+1)^2,.... This paper examines the conjecture made by Postnikov which involves examining the divisibility of L_n by powers of 3. This project gives an upper bound of 2*3^(2r-7) on the period of L_n modulo 3^r, which supports Postnikov's conjecture that this period is 2*3^(r-3). The results are proven by representing L_n using combinatorial structures called Dyck paths. Dyck paths of length n are broken into pieces using a process called partial flat path decomposition. This classifies paths according to the location of the steps corresponding to the weights divisible by 3^2 or the weight 1. Properties of partial flat paths are proven and this knowledge combined with the use of mathematical tools, specifically generating functions, lead to the main result.

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