Matrix Generalizations of the Euler Function

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The aim of my work is to study the matrix generalizations of the well-known Euler function. Recall, that the Euler function maps natural number m to a number of invertible elements from set Z_m of integers modulo m. Instead of Z_m I consider a set of matrices 2 by 2 with elements from Z_m. Then we define the Euler matrix function as a function that maps m to a number of invertible matrices from our set. It is natural and important to study the properties of the Euler matrix function, as it helps to generalize certain features and properties of natural numbers for matrices. Moreover, invertible matrices are often used in different areas of mathematics (e.g. in Gauss theory of quadratic binary forms) and its applications (e.g. in cryptography --- the Hill cipher). The most important results obtained in my work are the following. 1.The multiplicativity of the Euler matrix function was proved. 2.The formula for the values of the Euler matrix function was found. 3.The asymptotic behaviour of the Euler matrix function was obtained. To study the properties of the Euler matrix function I use methods from such different areas of mathematics as number theory, combinatorics, linear algebra and probability theory. The results of this research are amazing: they contain such fundamental mathematical objects as Riemann zeta function and infinite product over all primes.

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