An Application of Polynomials to Generate Pythagorean Triples

Marrero-Soto, Faviola

Pythagorean triples are derived from the Pythagorean Theorem, which states that three positive integer numbers a, b, c, representing the lengths of the sides and hypotenuse of a right triangle satisfy that the square of the hypotenuse is equal to the sum of the squares of the sides ($a^2 + b^2 = c^2$). Methods for generating Pythagorean triples have been known for about 2000 years. There are methods to find Pythagorean triples that use the Fibonacci sequence. After evaluating the existing methods to create Pythagorean triples the following question arises: Is there a method to generate Pythagorean triples using a polynomial? The hypothesis is: " If a method using a polynomial is established, then it will generate Pythagorean triples", which turned out to be true. Upon completion of this research work, a method to generate Pythagorean triples with a polynomial with integer coefficients was developed. This paper presents this direct method for generating Pythagorean triples, both primitive and non-primitive. The method developed in this research uses the fact that for any given positive integer in the variable of the polynomial, and a positive integer number greater or equal than two in the exponent, an infinite number of Pythagorean triples can be generated. This polynomial; 2x^n+2x^(n-1)+x^(n-2) where n is greater or equal to 2 and x is greater than 0, is obtained by a generalization of the method of Fibonacci sequences. The new method is innovative and provides an efficient way to generate Pythagorean triples.

Awards Won: