Characterizing the n-Division Points of Genus-0 Curves through Straightedge and Compass Constructions

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One of the most famous mathematical problems is that of constructing figures with the aid of an unmarked straightedge and compass. A subset of this field concerns the regular polygons that can be constructed with a compass and straightedge. This can be generalized to consider the n-division points of all curves. Little work has been done in this field, apart from two major theorems, the Gauss-Wantzel Theorem and Abel's Theorem on the Lemniscate that characterize the values of n for which the n-division points of the circle and lemniscate, respectively, are constructible. This research project examined three major problems in this field: determining a closed form solution for the regular polygons that can be constructed with a straightedge, compass, and trisector; finding the values of n such that the n-division points of the tricuspoid can be constructed with compass and straightedge; and seeking a generalization of Abel's Theorem on the Lemniscate to the entire family of Serret curves. Two major mathematical fields were used to algebraically represent the problems: field theory (for constructible numbers) and the theory of elliptic curves (for n-division points). Three major mathematical tools were used in the proofs of the theorems: Galois theory, algebraic geometry, and complex analysis. A closed-form solution for the values of n for which the n-division points of a circle can be constructed with a compass, straightedge, and trisector was found, a theorem was proved that for all integer n, the n-division points of the tricuspoid curve are constructible, and it was determined that with a compass and a straightedge, arbitrary arc lengths on any Serret Curve can be added, subtracted, and multiplied.

Awards Won:

Fourth Award of $500

American Mathematical Society: First Award of $2,000