Odd Dunkl Operators and nilHecke Algebras

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Symmetric functions appear in many areas of mathematics and physics, including enumerative combinatorics, the representation theory of symmetric groups, statistical mechanics, and the quantum statistics of ideal gases. When these functions commute, they give rise to a nilHecke algebra useful for studying Dunkl operators, used in physics to study the quantum fractional Hall problem. In this project, we use representation theory to study certain noncommutative symmetric functions that have a multiplication rule reminiscent of the classical cross product. The motivation for studying these “odd” symmetric functions comes from the categorification of quantum groups and odd Khovanov homology, which has applications in graph theory and topology. We contribute towards the representation-theoretic study of odd Khovanov homology by studying odd Dunkl operators in the setting of the odd nilHecke algebra. Specifically, we show that odd divided difference operators can be used to construct odd Dunkl operators, which we use to give a representation of a Lie group on the algebra of skew polynomials and evaluate the odd Dunkl Laplacian. We then consider a more general noncommutative structure in which ab=qba, and introduce new algebras which act on q-symmetric polynomials. We describe such algebras for all previously unstudied values of q. We conclude by generalizing a diagrammatic method for studying q-symmetric polynomials from the perspective of bialgebras.

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