# Generating Functions of the Free Generators of Some Submagmas of the Free Omega Magma and Planar Trees

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The concept of a free object is one that relates to all kinds of algebraic structures, i.e. free groups, free algebras. In this project we investigate one of the most fundamental free objects – the free Omega magma. Its universality is demonstrated by the lack of imposed standard properties such as commutativity, associativity, invertibility or existence of an identity. In addition, the Omega stands for a set of operations which can be from any arity, excluding zero and one. The free Omega magma can be described in terms of labeled reduced planar rooted trees. This yields a natural grading and allows examining the generating function of the free Omega magma for different sets Omega. In an article from 2008 V. Drensky and R. Holtkamp study the free Omega magma generated by one element when Omega contains only one binary operation. They consider the submagma of the elements of even degree (defined by the grading) and succeed in finding the generating function of its free generators. In this paper we extend their problem to bigger Omega sets (all finite and some infinite ones) and, moreover, investigate submagmas consisting of the elements having a degree divisible by a given positive integer s. We present a method which finds the equations satisfied by the generating functions of the free generators of these submagmas and in some cases obtain closed formulas for these generating functions as well. In addition, we prove that for every finite set Omega and every positive integer s, the generating function of the free generators is always irrational, but still algebraic. By doing this, we make a step forward in the analysis of this fundamental mathematical structure.

**Awards Won:**
Third Award of $1,000
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