Why Are We 3D: Modelling the N-Dimensional Electron Orbitals

Muntz, Benjamin (School: H.C. Orsted Gymnasiet i Lyngby)

One of the great unanswered questions in fundamental theoretical physics is "Why are we 3D?". In this research, I study the electron orbitals and their chemical properties to see, what would happen if they were introduced to a multidimensional space. The hyperspherical harmonics (HH) and radial wavefunctions are solved using the N-dimensional time-independent Schrödinger equation by means of recursive Legendre polynomials and Laplace transform. The HH lead only to converging wavefunctions for only some quantum numbers, meaning a strict decrease in allowed quantized states for N>3. Using Python, the HH were projected into 3D by taking cross-sections, reavealing that the orbitals take different shapes depending on the angle of projection. Furthermore, for increasing N, the radial probability distribution (RPD) tends towards the 3D RPD of the subshell with lowest energy, and converges faster for lower shells. Hence, the expectation value tends towards n^2*a_0 for large N. The electron degeneracy in the subshells in N dimensions were derived from quantum number restricted orbital degeneracy and the spin degeneracy of the N-space. Due to accidental isomorphisms between the Spin(N) and classical Lie groups for N<7, this could be determined, but not generalized. 4D, 5D and 6D periodic tables were drawn, illustrating that for sufficiently large N, all current elements could have their electrons contained in the 1s orbital, with a radial expectation value approximately equal to the Bohr radius. It is concluded that in spaces with N>3 dimensions, there is an immediate decrease in orbital degeneracy and ionization energy. Not only does this cause less stable atoms, but also a decrease in symmetry for higher N. This might explain from an atomic physics perspective, why we are 3D.

Awards Won:
Second Award of $2,000