The Generalization of Finsler-Hadwiger's Inequality on General Hyperbolic Triangles and Its Applications

Liu, Yingxin (School: Beijing No.161 High School)

Finsler-Hadwiger's inequality is a classical inequality, which is a stronger version of Weitzenboeck inequality. It connects side lengths and area of Euclidean triangles. The works on Finsler-Hadwiger's inequality are mostly about the sharpness. Also, it has been generalized to Euclidean n-space. In this project we give the integrated generalization of Finsler-Hadwiger’s inequality on general hyperbolic triangles by a general term formula. In order to explain the characteristic of the generalized inequality well and by considering its applications, we give the estimation on the perimeter of hyperbolic the pair of pants when its seams are very small (it includes Good Pants). Moreover, we give an example in estimating the perimeter of hyperbolic pentagons with four right angles. Furthermore, we decide to generalize Finsler-Hadwiger’s inequality to 3-dimension. But the exist volume formulas are rather far from the elementary formula and the symmetry of simplex seems to have more effects on its geometric data. In other words, what we know about the high dimensional volume formula is fairly few. It’s difficult and seems not to be a proper time to generalize. Instead, it may be better to replace the “volume” by the simplex’s perimeter or Gram matrix in this situation. Best to our knowledge, the generalized inequalities for non-Euclidean polygons have not been studied before, neither have the perimeter estimates by a synthetic geometrical approach. Our results seem to be the first attempt in those directions.

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