# A Fine Classification of Second Minimal Odd Orbits

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This paper presents the solution to the open problem of the fine classification of second minimal odd periodic orbits of continuous endomorphisms on the real line. It is rigorously proven that second minimal odd orbits (of any length 2k+1, where k>2), either have a Stefan structure, like that of a minimal odd orbit, or have one of 4k-3 types, each characterized with its own cyclic permutation and directed graph of transitions, unique up to an inverse. The result is applied to the overarching problem of deriving a universal law of distribution for periodic windows within the chaotic regime of the bifurcation diagram of the one-parameter family of unimodal maps. This paper adds to the growing evidence that a universal pattern of distribution can be revealed by fixing the maximum number of appearances for periodic windows, and comparing the topological structures of iterative orbits to the topological structure of the P-Linearization of Cyclic Permutations. By proving a fine classification of second minimal odd orbits, similar to the fine classification of minimal odd orbits by Stefan in 1977, this paper has a tremendous impact in the field of Chaos Theory, offering a deeper understanding of the universal nature of periodic orbits that appear in the iterative functions used in applied mathematical fields such as Population Dynamics, Weather Prediction, and Fluid Turbulence.

**Awards Won:**
Intel ISEF Best of Category Award of $5,000
First Award of $5,000
Dudley R. Herschbach SIYSS Award
American Mathematical Society: Third Award of $500