On the Maximum Number of Non-Intersecting Diagonals in Unit Squares Filling an n*n Grid

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Consider a square of integer side N divided into unit squares. The opposite vertices of some unit squares are connected with diagonal lines so that none of the diagonals have common points. The aim of the research is to estimate bounds on the largest possible number of diagonals subject to the assumptions made above. In the first part of the research we assume that N is even. In this case, we prove that lower bound and upper bound coincide. The conclusion is that the maximum number of diagonals equals N(N + 1)/2. The second part focuses on lower bound for odd numbers. The examples for conjecturally optimal positions of diagonals in such squares are given. The third part is the most important one. In this part an upper bound for odd N is considered. An assumption about the structure of optimal configurations of diagonals is made and an upper bound is deduced from this assumption. This bound coincides with the lower bound given in part two.

Awards Won:
Second Award of $2,000
Mu Alpha Theta, National High School and Two-Year College Mathematics Honor Society: First Award of $ 1,500