Using Lyapunov Exponents to Differentiate Between Randomness and Pseudorandomness

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The ability to tell between truly random numbers and pseudorandom numbers generated by computers is important in many scientific fields and real life applications. Pseudorandom number generators are based on chaotic functions, functions where two very close inputs give wildly different outputs. The largest Lyapunov exponent (LLE) is a measure of the rate at which two points in the domain of a function separate in the range, called divergence. A positive LLE means the data is diverging and is an indicator of chaos. A positive LLE could show that a function is not truly random by showing that it is based on a chaotic function. This idea was used in an experiment. It was hypothesized that random data would output a negative LLE, and pseudorandom data would output a positive LLE. Rosenstein's algorithm was used to estimate the LLE. Inputting single digits from random digits of pi and pseudorandom numbers both gave positive LLEs. In a second experiment, large decimal numbers from random atmospheric noise were compared to numbers from a chaotic function called the logistic map. These gave negative LLEs for random data and positive LLEs for chaotic data, supporting the hypothesis. The reason the first experiment didn't support the hypothesis is most likely because single digits were inputted instead of large decimal numbers, resulting in a small sample size. It can be concluded that this system could be used to distinguish between random and pseudorandom numbers when large amounts of data are available.

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