Discrete Derivatives of Random Matrix Models and the Gaussian Free Field

Goel, Gopal (School: Krishna Homeschool)

A random matrix is a matrix whose entries are given by random variables. The spectral statistics of high dimensional random matrices have shown deep connections with physics over the past century. It is known to arise in the study of Coulomb gases, such as the Gaussian Orthogonal Ensemble and the Gaussian Unitary Ensemble for 1 and 2 dimensional Coulomb gases respectively. In general, one can construct a beta-matrix model corresponding to beta-dimensional Coulomb gases where beta interpolates between integers. The asymptotic behavior in high dimensions of these families of random matrices can be described by the Gaussian free field. We consider the question of how robust this convergence is under differentiation. We show, for each beta, that the discrete derivative of the beta-matrix model converges to the derivative of the Gaussian free field. Our approach extends what was previously known only for beta = 1, 2 to arbitrary beta and simplifies many of the analytic proofs.

Awards Won:

Fourth Award of $500

American Mathematical Society: Second Award of $1,000